Bottomonium Spectrum with Screened Potential

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As a sister work of Ref. [1], we incorporate the color-screening effect due to light quark pair creation into the heavy quark-antiquark potential, and investigate the effects of screened potential on the spectrum of bottomonium. We calculate the masses, electromagnetic decays, and E1 transitions of bottomonium states. We find that the fine splittings between χ_{bJ} (J=0,1,2) states are in good agreement with experimental data, and the E1 transition rates of $\Upsilon(2S) \to \gamma\chi_{bJ}(1P)$ and $\Upsilon(3S) \to \gamma\chi_{bJ}(2P)$ (J=0,1,2) all agree with data within experimental errors. In particular, the mass of $\Upsilon(6S)$ is lowered down to match that of the $\Upsilon(11020)$, which is smaller than the predictions of the linear potential models by more than 100 MeV. Comparison between charmonium and bottomonium in some related problems is also discussed.

Key Words: Color-screening effect, Bottomnium

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I. INTRODUCTION

Potential models have been successful in describing the spectra below the open-flavor thresholds for both charmonia and bottomonia. However, it is well-known that these quenched potential models, which incorporates a Coulomb term at short distances and a linear confining potential at large distances [2, 3, 4], will overestimate the masses of heavy quarkonia above the open-flavor thresholds. Some distinct examples are the $\chi_{c2}'/Z(3930)$ in the charmonium system and the $\Upsilon(6S)$ in the bottomonium system, of which the observed masses are about 50 and $90\,MeV$ respectively lower than that predicted by the typical relativized potential model of Godfrey and Isgur [4], and even $120\,MeV$ lower for $\Upsilon(6S)$ than the prediction of the Cornell model [2, 3].

This is probably because, the linear potential, which is expected to be dominant at large distances, is screened or softened by the vacuum polarization effect of the dynamical light quark pairs [5]. This screening or string breaking effect has been demonstrated indirectly by the investigation of the mixing of static heavy quark-antiquark ($Q\bar{Q}$) string with a static heavy-light meson-antimeson ($Q\bar{q}$-$\bar{Q}q$) system in the $n_f = 2$ lattice QCD calculations [6], and has also been implied recently by the calculations within some holographic QCD models [7].

However, since the simulations of lattice QCD still have large uncertainties and difficulties in handling higher excited states, it should be useful to improve the potential model itself to incorporate the screening effect and compare the model predictions with experimental data, as a phenomenological way to investigate the screening effects on heavy quarkonium spectrum.

Such screened potential models [8, 9, 10, 11] were proposed many years ago in the study of heavy quarkonium and heavy flavor mesons, as well as light hadrons[12]. The main feature of these screened potential models in the spectrum is that the masses of the higher excited states are lowered.

In recent years the screened potential models have again been used to investigate the heavy quarkonium spectrum and leptonic decay widths [13]. In Ref.[1] we have reinvestigated the charmonium spectrum within the screened potential model suggested by Chao and Liu [8] and assigned some newly discovered charmonium-like resonances as conventional higher charmonium states.

On the experimental side, aside from abundant resonances discovered recently in the charmonium region, progress in the bottomium region has also been made. The $\Upsilon(1D)$ was observed by CLEO collaboration [14] in 2004 and the η_b was observed by BaBar collaboration [15] in 2008. One may expect more bottomium states will be observed in the future by BaBar, Belle and CLEO. So it is important to reinvestigate the bottomonium system within the screened potential model.

In this paper, as a sister work of [1], we calculate the mass spectrum and electromagnetic decay and transition rates of bottomonium especially the higher bottomonium using a non-relativistic Schrödinger equation with the Coulomb potential plus a screened linear potential, which is nearly the same as that in [1]. The model predictions are similar to that of [1] for charmonium. The mass of $\Upsilon(6S)$ is lowered to be consistent with its experimental value.

In the following, we first introduce the screened potential model in Sec.II, and then study the mass spectrum and decay and transition processes of bottomonia in Sec. III. In Sec. IV we will discuss some features of our result for the bottomonium states. A summary will be given in Sec. V.

II. THE SCREENED POTENTIAL MODEL

As a minimal model describing the bottomonium spectrum we use a non-relativistic potential model with the screening effect being considered as in [1]. We use a potential as

$$V_{scr}(r) = V_V(r) + V_S(r), \tag{1}$$

$$V_V(r) = -\frac{4}{3}\frac{\alpha_C}{r}, \tag{2}$$

$$V_S(r) = \lambda(\frac{1-e^{-\mu r}}{\mu}) + C. \tag{3}$$

Here μ is the screening factor which makes the long-range scalar potential of $V_{scr}(r)$ become flat when $r \gg \frac{1}{\mu}$ and still linearly rising when $r \ll \frac{1}{\mu}$, and λ is the linear potential slope (the string tension), which is taken to be the same as for charmonium[1]. $V_V(r)$ represents the vector-like one-gluon exchange potential, α_C is the coefficient of the Coulomb potential. C is a constant related to the normalization of energy levels of the $Q\bar{Q}$ system.

The spin-dependent interactions include three parts as follows. The spin-spin contact hyperfine interaction is

$$H_{SS} = \frac{32\pi\alpha_C}{9m_b^2}\,\tilde{\delta}_\sigma(r)\,\vec{S}_b\cdot\vec{S}_{\bar{b}}\,, \tag{4}$$

where $\tilde{\delta}_\sigma(r)$ is usually taken to be $\delta(\vec{r})$ in nonrelativistic potential models, but here we take $\tilde{\delta}_\sigma(r) = (\sigma/\sqrt{\pi})^3\,e^{-\sigma^2 r^2}$ as in Ref.[16] since it is an artifact of an $O(v_b^2/c^2)$ expansion of the T-matrix [17] in a range comparable to $1/m_b$.

The spin-orbit term and the tensor term take the common forms

$$H_{LS} = \frac{1}{2m_b^2 r}(3V_V'(r) - V_S'(r))\vec{L}\cdot\vec{S}, \tag{5}$$

and

$$H_T = \frac{1}{12m_b^2}(\frac{1}{r}V_V'(r) - V_V''(r))T. \tag{6}$$

These spin-dependent interactions are dealt with perturbatively. They are diagonal in a $|J,L,S>$ basis with the matrix elements

$$<\vec{S}_b\cdot\vec{S}_{\bar{b}}> = \frac{1}{2}S^2 - \frac{3}{4}, \tag{7}$$

$$<\vec{L}\cdot\vec{S}> = [J(J+1) - L(L+1) - S(S+1)]/2 \tag{8}$$

and the tensor operator T has nonvanishing diagonal matrix elements only between $L>0$ spin-triplet states, which are

$$<^3L_J|T|^3L_J> = \begin{cases} -\frac{L}{6(2L+3)}\,, & J = L+1 \\ \frac{1}{6}\,, & J = L \\ -\frac{(L+1)}{6(2L-1)}\,, & J = L-1. \end{cases} \tag{9}$$

For the model parameters, we take

$$\begin{aligned} \alpha_C = 0.37, \alpha_S = 0.18, \\ \mu = 0.056\,GeV, C = 0.677\,GeV, \\ m_b = 4.4\,GeV, \sigma = 3.3\,GeV, \lambda = 0.21 GeV^2 \end{aligned} \tag{10}$$

where $\alpha_C \approx \alpha_s(m_b v_b)$ and $\alpha_S \approx \alpha_s(2m_b)$ are essentially the strong coupling constants but at different scales. The former is for large distances and used to determine the spectrum while the latter is for short-distances and used for QCD radiative corrections in bottomonium decays (see below in next section).

Here μ is the characteristic scale for color screening, and $1/\mu$ is about $3.5\,fm$, implying that at distances larger than $1/\mu$ the static color source in the $b\bar{b}$ system gradually becomes neutralized by the produced light quark pair, and string breaking emerges. Note that here μ is smaller than that of charmonium in [1], where $\mu = 0.0979\,GeV$ corresponding to $2\,fm$. In Sec.IV we will discuss the reason for the difference in μ between $b\bar{b}$ and $c\bar{c}$ systems.

With these values of the parameters for the potential, we can calculate the spectrum of bottomonium. The results are shown in Table I. For comparison, we also list the experimental values [18] and those predicted by the linear potential model [4] in Table I.

III. SOME DECAY PROCESSES

A. Leptonic decays

The electronic decay width of the vector meson is given by the Van Royen-Weisskopf formula [19] with QCD radiative corrections taken into account [20].

$$\Gamma_{ee}(nS) = \frac{4\alpha^2 e_b^2}{M_{nS}^2}|R_{nS}(0)|^2(1 - \frac{16}{3}\frac{\alpha_S}{\pi}), \tag{11}$$

$$\Gamma_{ee}(nD) = \frac{25\alpha^2 e_b^2}{2M_{nD}^2 m_b^4}|R_{nD}''(0)|^2(1 - \frac{16}{3}\frac{\alpha_S}{\pi}), \tag{12}$$

where $M_{nS}(M_{nD})$ is the mass for $nS(nD)$, $e_b = \frac{1}{3}$ is the b quark charge, α is the fine structure constant, $R_{nS}(0)$ is the radial S wave-function at the origin, and $R_{nD}''(0)$ is the second derivative of the radial D wave-function at the origin.

With the chosen parameters (10), we get the results that are listed in table II. We also list other two groups' results [21, 22] for comparison. We can see that our results are consistent with the experimental data.

B. Two-photon decays

In the nonrelativistic limit, the two-photon decay widths of the $^1S_0$, $^3P_0$, and $^3P_2$ states can be written

as [23]

$$\Gamma^{NR}(^1S_0 \to \gamma\gamma) \; = \; \frac{3\alpha^2 e_b^4 |R_{nS}(0)|^2}{m_b^2}\,, \qquad (13)$$

$$\Gamma^{NR}(^3P_0 \to \gamma\gamma) \; = \; \frac{27\alpha^2 e_b^4 |R'_{nP}(0)|^2}{m_b^4}\,, \qquad (14)$$

$$\Gamma^{NR}(^3P_2 \to \gamma\gamma) \; = \; \frac{36\alpha^2 e_b^4 |R'_{nP}(0)|^2}{5m_b^4}. \qquad (15)$$

The first-order QCD radiative corrections to the two-photon decay rates can be accounted for as [23]

$$\Gamma(^1S_0 \to \gamma\gamma) \; = \; \Gamma^{NR}(^1S_0 \to \gamma\gamma)[1 + \frac{\alpha_S}{\pi}(\frac{\pi^2}{3} - \frac{20}{3})]\,, \qquad (16)$$

$$\Gamma(^3P_0 \to \gamma\gamma) \; = \; \Gamma^{\mathrm{NR}}(^3P_0 \to \gamma\gamma)[1 + \frac{\alpha_S}{\pi}(\frac{\pi^2}{3} - \frac{28}{9})]\,, \qquad (17)$$

$$\Gamma(^3P_2 \to \gamma\gamma) \; = \; \Gamma^{\mathrm{NR}}(^3P_2 \to \gamma\gamma)[1 - \frac{16}{3}\frac{\alpha_S}{\pi}]\,. \qquad (18)$$

We can see that $\Gamma(^1S_0 \to \gamma\gamma) \propto |R_{nS}(0)|^2$, which are sensitive to the details of the potential near the origin. So we take

$$\Gamma(^1S_0 \to \gamma\gamma) \longrightarrow \frac{\Gamma(^1S_0 \to \gamma\gamma)}{\Gamma_{ee}(nS)} \Gamma_{ee}^{expt}(nS) \qquad (19)$$

to eliminate this uncertainty.

In the nonrelativistic limit, we can also replace m_b by M/2, where M is the mass of the corresponding bottomonium state. The results are listed in Table III. Predictions of some other models (see Refs. [4, 24, 25, 26, 27, 28, 29]) are listed for comparison. We can see our results are a bit larger than most models but are consistent with Refs. [28, 29].

C. E1 transitions

For the E1 transitions within the bottomonium system, we use the formula of Ref. [30]:

$$\begin{aligned}&\Gamma_{\mathrm{E1}}(\mathrm{n}\,{}^{2\mathrm{S}+1}\mathrm{L}_{\mathrm{J}} \to \mathrm{n}'\,{}^{2\mathrm{S}'+1}\mathrm{L}'_{\mathrm{J}'} + \gamma) \\ &= \frac{4}{3}\, C_{fi}\, \delta_{\mathrm{SS}'}\, e_b^2\, \alpha\, |\, \langle f |\, r \,| i \rangle \,|^2\, \mathrm{E}_\gamma^3 \end{aligned} \qquad (20)$$

where E_γ is the emitted photon energy.

The spatial matrix element

$$< f|r|i > = \int_0^\infty R_f(r) R_i(r) r^3 dr\,, \qquad (21)$$

involves the initial and final state radial wave functions, and the angular matrix element C_{fi} is

$$C_{fi} = \max(\mathrm{L},\ \mathrm{L}')\ (2\mathrm{J}' + 1) \begin{Bmatrix} \mathrm{L}' & \mathrm{J}' & \mathrm{S} \\ \mathrm{J} & \mathrm{L} & 1 \end{Bmatrix}^2 . \qquad (22)$$

Our results are listed in Table VI. The widths calculated by the zeroth-order wave functions are marked by SNR_0 and those by the first-order relativistically corrected wave functions are marked by SNR_1.

For the first-order relativistic corrections to the wave functions, we include the spin-dependent part of (4),(5),(6) and the spin-independent part as [31]

$$\begin{aligned} H_{SI} \; = \; & -\frac{\vec{P}^4}{4m_b^3} + \frac{1}{4m_b^2} \bigtriangledown^2 V_V(r) \\ & -\frac{1}{2m_b^2} \left\{ \left\{ \vec{P}_1 \cdot V_V(r) \Im \cdot \vec{P}_2 \right\} \right\} \\ & +\frac{1}{2m_b^2} \left\{ \left\{ \vec{P}_1 \cdot \vec{r} \frac{V'_V(r)}{r} \vec{r} \cdot \vec{P}_2 \right\} \right\}, \end{aligned} \qquad (23)$$

where $\vec{P}_1$ and $\vec{P}_2$ are the momenta of b and $\bar{b}$ quarks in the rest frame of bottomonium, respectively, which satisfy $\vec{P}_1 = -\vec{P}_2 = \vec{P}$, $\Im$ is the unit second-order tensor, and $\{\{\quad\}\}$ is the Gromes's notation

$$\{\{\vec{A}\cdot\Re\cdot\vec{B}\}\} = \frac{1}{4}(\vec{A}\vec{B} : \Re + \vec{A}\cdot\Re\vec{B} + \vec{B}\cdot\Re\vec{A} + \Re : \vec{A}\vec{B}), \qquad (24)$$

where $\Re$ is any second-order tensor.

Note that we do not include the contributions from the scalar potential in H_{SI} since it is still unclear how to deal with the spin-independent corrections arising from the scalar potential theoretically.

We also list in Table VI the results of Ref.[30] which uses a potential obtained from the inverse-scattering method for comparison.

Both the SNR_0 and Ref.[30] results of E1 transitions are larger than most of the experimental values, but we see that in SNR_1 the predicted widths get decreased and fit the experimental values quite well as long as the first-order relativistic corrections to the wave functions are taken into account.

IV. DISCUSSIONS

A. $\Upsilon(11020)$

$\Upsilon(11020)$, the candidate of $\Upsilon(6S)$, was observed in e^+e^- annihilation in 1985 [32, 33]. Its PDG mass and full width [18] are

$$\begin{aligned} M &= 11019 \pm 8\, MeV, \\ \Gamma &= 79 \pm 16\, MeV. \end{aligned} \qquad (25)$$

Recently, the BaBar collaboration [34] has remeasured the $e^+e^- \to b\bar{b}$ cross section by an energy scan in the range of $10.54\, GeV$ to $11.20\, GeV$ and get the mass and full width as

$$\begin{aligned} M &= 10996 \pm 2\, MeV, \\ \Gamma &= 37 \pm 3\, MeV. \end{aligned} \qquad (26)$$

Despite of the discrepancy in the mass and full width given by BaBar and PDG, the observed mass is much

smaller than that predicted by the linear potential models. For example, the Cornell model [2, 3] predicted $11.14\,GeV$, which is $121(144)\,MeV$ larger than the experimental value of PDG(BaBar), and the modified Cornell model [35] gives $11.113\,GeV$, which is $104(127)\,MeV$ larger than the experimental value of PDG(BaBar). The relativized potential model of Godfrey and Isgur [4] gives $11.10\,GeV$, which is $91(114)\,MeV$ larger than the experimental value of PDG(BaBar).

Evidently, the mass of $\Upsilon(6S)$ is overestimated by the quenched potential models by more than 100 MeV. If we take the screening effect into account, we find, in our model, the mass of $\Upsilon(6S)$ to be $11.023\,GeV$, which is very close to the observed value of PDG(BaBar). The consistence of our predicted mass with the experimental value of $\Upsilon(11020)$ indicates the significance of the screening effect on higher excited bottomonium states.

B. Hyperfine and Fine Splittings

We use Eq.(4) to calculate the hyperfine splittings between $\Upsilon(nS)$ and $\eta_b(nS)$, where $\tilde{\delta}_\sigma(r)$ is taken to be $\tilde{\delta}_\sigma(r) = (\sigma/\sqrt{\pi})^3\, e^{-\sigma^2 r^2}$ as in Ref.[16]. σ has a magnitude of order m_Q and it represents some relativistic smearing effects[16].

We choose the observed splitting between J/ψ and η_c as input to determine σ for charmonium and have obtained a good fit for the $\psi(3686)$-$\eta_c(3637)$ splitting [1]). Here we use the observed η_b [15] and $\Upsilon(1S)$ masses to determine σ for bottomonium, and find σ to be $3.3\,GeV$ in our model [see (10)]. The hyperfine splittings for charmonium and bottomonium systems are listed in Tab.V. In comparison, we also list the results of Ref.[36], which uses the Buchmüller-Tye potential and Ref.[4], which uses a relativized funnel potential.

We also list the results of the fine splittings, which are calculated by using (5) and (6), between P-wave multiplets for both charmonium and bottomonium in the same table. We can see that our results fit the experimental values quite well and are compatible with Ref.[36] and Ref.[4].

C. E1 transitions

We have calculated the E1 transition widths for bottomonium using the zeroth-order wave functions, which are marked by SNR_0, and the first-order relativistically corrected wave functions, which are marked by SNR_1. The results are listed in Table VI, along with the results from the potential model in Ref.[30], in which the potential is determined by the inverse-scattering method, for comparison.

We find our results are compatible with experimental values and Ref.[30] for most channels. Relativistic corrections to the wave functions tend to reduce the E1 transition widths for most channels and give better fit with experimental values. Note that for the $\Upsilon(3S) \to \gamma\chi_{b0}$ transition our result, $0.07(0.05)\,KeV$ with the zeroth(first)-order wave functions, is in agreement with the experimental value $0.061 \pm 0.023\,KeV$, while that of Ref.[30]($0.007\,KeV$) is too small.

But our calculated transition widths for $\Upsilon(3S) \to \gamma\chi_{bJ}(J=1,2)$ are too large as compared with experimental data. These may indicate that for the radially suppressed E1 transition widths (e.g. $\Upsilon(3S) \to \gamma\chi_{bJ}(J=0,1,2)$) the theoretical values are very sensitive to model details, and further improvement for the model and the calculation is needed.

D. Screening parameter μ

We find the screening parameter μ, which represents the energy scale related to the creation of the $Q\bar{q}$ and $\bar{Q}q$ pair or the distance when that beyond $r \sim 1/\mu$ the screening effect becomes important, is smaller for bottomonium ($\mu = 0.056\,GeV$) than that for charmonium ($\mu = 0.0979\,GeV$)[1], if we try to fit the bottomonium spectrum. We need to understand this difference of μ between the $b\bar{b}$ and $c\bar{c}$ systems.

It is known that the string breaking is due to the creation of light quark pairs, i.e., the formation of $Q\bar{q}$-$\bar{Q}q$ mesons. Note that for the $b\bar{b}$ system the energy difference between the $b\bar{b}$ bound state $\Upsilon(1S)$ and the open bottom threshold of $B\bar{B}$ meson pair is 1.1 GeV, whereas for the $c\bar{c}$ system the energy difference between the $c\bar{c}$ bound state J/ψ and the open charm threshold of $D\bar{D}$ meson pair is only 0.63 GeV. This implies that for the $b\bar{b}$ system more energy needs to be stored (or equivalently a longer flux tube is neded) before the string breaking occurs than that for the $c\bar{c}$ system.

V. SUMMARY AND CONCLUSIONS

In this paper, as a sister work of [1], we incorporate the color-screening (string breaking) effect due to light quark pair creation into the heavy quark-antiquark long-range confinement potential, and investigate the effects of screened potential on the spectrum of bottomonium. We calculate the masses, electromagnetic decays, and E1 transitions of bottomonium states in the nonrelativistic screened potential model.

We find that the screening parameter μ is smaller for bottomonium than that for charmonium if we try to fit the bottomonium spectrum, and this may be understood as due to the difference between $b\bar{b}$ and $c\bar{c}$ systems in the energy to be stored before the string breaking occurs. The masses predicted in the screened potential model are considerably lower for higher bottomonium states,

compared with the unscreened potential model. Especially, the mass of $\Upsilon(6S)$ is lowered down to match that of $\Upsilon(11020)$, whereas the linear potential model predictions are more than 100 MeV higher than the experimental value. The fine splittings of P-wave bottomonium states, and E1 transition rates and leptonic decay widths are found to be compatible with experimental data within errors.

We hope our investigation for the bottomonium system with screened potential model will be useful in the future study of bottomonium physics.

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TABLE I: Experimental and theoretical mass spectrum of bottomonium states. The experimental masses are PDG [18] averages. The masses are in units of MeV except for Ref.[4] which is in GeV. The averaged radiuses are in units of fm. The results of our screened potential model are shown in comparison with that of Ref.[4].

State		Expt.	Theor. of ours		Theor. of Ref [4]
			Mass	$\langle r^2\rangle^{\frac{1}{2}}$	Mass
1S	$\Upsilon(1^3S_1)$	9460.30 ± 0.26	9460	0.23	9.46
	$\eta_b(1^1S_0)$	$9388.9^{+3.1}_{-2.3} \pm 2.7$	9389		9.40
2S	$\Upsilon(2^3S_1)$	10023.26 ± 0.31	10016	0.52	10.00
	$\eta_b'(2^1S_0)$		9987		9.98
3S	$\Upsilon(3^3S_1)$	10355.2 ± 0.5	10351	0.78	10.35
	$\eta_b(3^1S_0)$		10330		10.34
4S	$\Upsilon(4^3S_1)$	10579.4 ± 1.2	10611	1.02	10.63
	$\eta_b(4^1S_0)$		10595		
5S	$\Upsilon(5^3S_1)$	10865 ± 8	10831	1.24	10.88
	$\eta_b(5^1S_0)$		10817		
6S	$\Upsilon(6^3S_1)$	11019 ± 8	11023	1.45	11.10
	$\eta_b(6^1S_0)$		11011		
7S	$\Upsilon(7^3S_1)$		11193	1.66	
	$\eta_b(7^1S_0)$		11183		
1P	$\chi_{b2}(1^3P_2)$	$9912.21 \pm 0.26 \pm 0.31$	9918	0.42	9.90
	$\chi_{b1}(1^3P_1)$	$9892.78 \pm 0.26 \pm 0.31$	9897		9.88
	$\chi_{b0}(1^3P_0)$	$9859.44 \pm 0.42 \pm 0.31$	9865		9.85
	$h_b(1^1P_1)$		9903		9.88
2P	$\chi_{b2}(2^3P_2)$	$10268.65 \pm 0.22 \pm 0.50$	10269	0.69	10.26
	$\chi_{b1}(2^3P_1)$	$10255.46 \pm 0.22 \pm 0.50$	10251		10.25
	$\chi_{b0}(2^3P_0)$	$10232.5 \pm 0.4 \pm 0.5$	10226		10.23
	$h_c(2^1P_1)$		10256		10.25
3P	$\chi_{b2}(3^3P_2)$		10540	0.93	
	$\chi_{b1}(3^3P_1)$		10524		
	$\chi_{b0}(3^3P_0)$		10502		
	$h_b(3^1P_1)$		10529		
4P	$\chi_{b2}(4^3P_2)$		10767	1.15	
	$\chi_{b1}(4^3P_1)$		10753		
	$\chi_{b0}(4^3P_0)$		10732		
	$h_b(4^1P_1)$		10757		
5P	$\chi_{b2}(5^3P_2)$		10965	1.37	
	$\chi_{b1}(5^3P_1)$		10951		
	$\chi_{b0}(5^3P_0)$		10933		
	$h_b(5^1P_1)$		10955		
1D	$\psi_3(1^3D_3)$		10156	0.57	10.16
	$\psi_2(1^3D_2)$	$10161 \pm 0.6 \pm 1.6$	10151		10.15
	$\psi(1^3D_1)$		10145		10.14
	$\eta_{c2}(1^1D_2)$		10152		10.15
2D	$\psi_3(2^3D_3)$		10442	0.82	10.45
	$\psi_2(2^3D_2)$		10438		10.45
	$\psi(2^3D_1)$		10432		10.44
	$\eta_{c2}(2^1D_2)$		10439		10.45
3D	$\psi_3(3^3D_3)$		10680	1.05	
	$\psi_2(3^3D_2)$		10676		
	$\psi(3^3D_1)$		10670		
	$\eta_{c2}(3^1D_2)$		10677		
4D	$\psi_3(4^3D_3)$		10886	1.27	
	$\psi_2(4^3D_2)$		10882		
	$\psi(3^4D_1)$		10877		
	$\eta_{c2}(4^1D_2)$		10883		
5D	$\psi_3(5^3D_3)$		11069	1.49	
	$\psi_2(5^3D_2)$		11065		
	$\psi(5^3D_1)$		11060		
	$\eta_{c2}(5^1D_2)$		11066		

TABLE II: Leptonic decay widths (in units of KeV) for bottomonium states in our screened potential model. The widths calculated with and without QCD corrections are marked by Γ_{ee} and Γ^0_{ee} respectively. The experimental values are taken from PDG [18]. Predictions of two other models[21, 22] are listed for comparison.

state	Γ^0_{ee}	Γ_{ee}	Ref.[21]	Ref.[22]	Exp[18]
$1^3S_1(9460)$	2.31	1.60	1.314	1.320	1.340 ± 0.018
$2^3S_1(10023)$	0.92	0.64	0.576	0.628	0.612 ± 0.011
$3^3S_1(10355)$	0.64	0.44	0.476	0.263	0.443 ± 0.008
$4^3S_1(10579)$	0.51	0.35	0.248	0.104	0.272 ± 0.029
$5^3S_1(10865)$	0.42	0.29	0.310	0.04	0.31 ± 0.07
$6^3S_1(11019)$	0.37	0.25			0.130 ± 0.030
$7^3S_1(11193)$	0.32	0.22			

TABLE III: Two-photon decay widths (in units of eV) of the pseudoscalar ($^1S_0$), scalar ($^3P_0$), and tensor ($^3P_2$) bottomonium states. Bottomonium masses are in units of MeV.

		Theory								Experiment
state	mass	Ref.[24]	Ref.[25]	Ref.[4]	Ref.[26]	Ref.[27]	Ref.[28]	Ref.[29]	Ours	PDG [18]
$\eta_b(1^1S_0)$	9389	350	220	214	266	192	460	460	527	
$\eta_b(2^1S_0)$	9987	150	110	121	95.0	116		200	263	
$\eta_b(3^1S_0)$	10330	100	84	90.6	67.9	93.5			172	
$\eta_b(4^1S_0)$	10595		71	75.5	56.3	81.8			105	
$\eta_b(5^1S_0)$	10817								121	
$\eta_b(6^1S_0)$	11011								50	
$\chi_{b0}(1^3P_0)$	9859	38	24	20.8	27.3	24.1	80	43	37	
$\chi_{b0}(2^3P_0)$	10233	29	26	22.7	26.9	27.3			37	
$\chi_{b0}(3^3P_0)$	10502								35	
$\chi_{b2}(1^3P_2)$	9912	8	5.6	5.14	2.56	6.45	8	7.4	6.6	
$\chi_{b2}(2^3P_2)$	10269	6	6.8	6.21	6.11	8.1			6.7	
$\chi_{b2}(3^3P_2)$	10540								6.4	

TABLE IV: E1 transition rates of bottomonium states in our screened potential model (those calculated by the zeroth-order wave functions are marked by SNR_0 and those by the first-order relativistically corrected wave functions are marked by SNR_1). We also list the results of one potential model, in which the potential is determined by the inverse-scattering method, for comparison [30].

state	Initial meson	Final meson	E_γ (MeV) Ref [30]	E_γ (MeV) $SNR_{0(1)}$	$\Gamma_{\rm thy}$ (keV) Ref [30]	$\Gamma_{\rm thy}$ (keV) SNR_0	$\Gamma_{\rm thy}$ (keV) SNR_1	$\Gamma_{\rm expt}$ (keV) PDG [18]
2S → 1P	$\Upsilon(2^3S_1)(10023)$	$\chi_{b2}(1^3P_2)$	110	110	2.14	2.62	2.46	2.29 ± 0.23
		$\chi_{b1}(1^3P_1)$	131	130	2.18	2.54	2.08	2.21 ± 0.22
		$\chi_{b0}(1^3P_0)$	162	163	1.39	1.67	1.11	1.22 ± 0.16
	$\eta_c(2^1S_0)(9389)$	$h_b(1^1P_1)$		83		6.10	5.57	
3S → 2P	$\Upsilon(3^3S_1)(10355)$	$\chi_{b2}(2^3P_2)$	87	86	2.78	3.23	3.04	2.66 ± 0.41
		$\chi_{b1}(2^3P_1)$	99	99	2.52	2.96	2.44	2.56 ± 0.34
		$\chi_{b0}(2^3P_0)$	124	122	1.65	1.83	1.23	1.20 ± 0.16
	$\eta_c(3^1S_0)(10330)$	$h_b(2^1P_1)$		74		11.0	10.1	
3S → 1P	$\Upsilon(3^3S_1)$	$\chi_{b2}(1^3P_2)$	433	434	0.025	0.25	1.26	$< 0.386 \pm 0.035$
		$\chi_{b1}(1^3P_1)$	453	452	0.017	0.17	0.14	$< 0.0345 \pm 0.0031$
		$\chi_{b0}(1^3P_0)$	484	484	0.007	0.07	0.05	0.061 ± 0.023
	$\eta_c(3^1S_0)$	$h_b(1^1P_1)$		418		1.24	5.68	
4S → 3P	$\Upsilon(4^3S_1)(10579)$	$\chi_{b2}(3^3P_2)$		40		0.55	0.52	
		$\chi_{b1}(3^3P_1)$		55		0.91	0.74	
		$\chi_{b0}(3^3P_0)$		77		0.82	0.54	
	$\eta_c(4^1S_0)(10595)$	$h_b(3^1P_1)$		67		14.3	12.9	
4S → 2P	$\Upsilon(4^3S_1)$	$\chi_{b2}(2^3P_2)$		306		0.14	0.56	
		$\chi_{b1}(2^3P_1)$		319		0.09	0.001	
		$\chi_{b0}(2^3P_0)$		341		0.04	0.21	
	$\eta_c(4^1S_0)$	$h_b(2^1P_1)$		334		0.95	2.16	
4S → 1P	$\Upsilon(4^3S_1)$	$\chi_{b2}(1^3P_2)$		646		0.15	0.86	
		$\chi_{b1}(1^3P_1)$		664		0.10	0.20	
		$\chi_{b0}(1^3P_0)$		695		0.04	0.001	
	$\eta_c(4^1S_0)$	$h_b(1^1P_1)$		669		0.90	5.64	
1P → 1S	$\chi_{b2}(1^3P_2)(9912)$	$\Upsilon(1^3S_1)(9460)$	443	442	37.8	38.2	32.6	
	$\chi_{b1}(1^3P_1)(9893)$		443	423	32.8	33.6	30.0	
	$\chi_{b0}(1^3P_0)(9859)$		392	391	26.1	26.6	24.3	
	$h_b(1^1P_1)(9903)$	$\eta_b(1^1S_0)(9389)$		501		55.8	36.3	
2P → 2S	$\chi_{b2}(2^3P_2)(10269)$	$\Upsilon(2^3S_1)(10023)$	242	243	18.7	18.8	14.2	
	$\chi_{b1}(2^3P_1)(10255)$		230	230	15.9	15.9	13.8	
	$\chi_{b0}(2^3P_0)(10233)$		205	207	11.3	11.7	11.6	
	$h_b(2^1P_1)(10256)$	$\eta_b(2^1S_0)(9987)$		266		24.7	15.3	
2P → 1S	$\chi_{b2}(2^3P_2)$	$\Upsilon(1^3S_1)$	777	777	9.75	13.0	12.5	
	$\chi_{b1}(2^3P_1)$		765	764	9.31	12.4	8.56	
	$\chi_{b0}(2^3P_0)$		742	743	8.48	11.4	4.50	
	$h_b(2^1P_1)$	$\eta_b(1^1S_0)$		831		15.9	18.0	
3P → 3S	$\chi_{b2}(3^3P_2)(10540)$	$\Upsilon(3^3S_1)$	170	183	12.1	15.6	11.1	
	$\chi_{b1}(3^3P_1)(10524)$		159	167	10.1	12.0	9.97	
	$\chi_{b0}(3^3P_0)(10502)$		144	146	7.46	7.88	7.67	
	$h_b(3^1P_1)(10529)$	$\eta_b(3^1S_0)$		196		19.2	11.6	
3P → 2S	$\chi_{b2}(3^3P_2)(10540)$	$\Upsilon(2^3S_1)$	491	504	3.78	6.00	6.89	
	$\chi_{b1}(3^3P_1)(10524)$		481	489	3.56	5.48	5.39	
	$\chi_{b0}(3^3P_0)(10502)$		466	468	3.24	4.80	3.67	
	$h_b(3^1P_1)(10529)$	$\eta_b(2^1S_0)$		528		6.89	10.3	
3P → 1S	$\chi_{b2}(3^3P_2)(10540)$	$\Upsilon(1^3S_1)$	1012	1024	3.80	7.09	6.76	
	$\chi_{b1}(3^3P_1)(10524)$		1003	1010	3.69	6.80	3.39	
	$\chi_{b0}(3^3P_0)(10502)$		989	990	3.54	6.41	0.86	
	$h_b(3^1P_1)(10529)$	$\eta_b(1^1S_0)$		1078		8.27	9.46	
2P → 1D	$\chi_{b2}(2^3P_2)$	$\Upsilon(1^3D_3)(10156)$	107	113	2.62	3.33	3.13	
		$\Upsilon(1^3D_2)(10151)$	112	117	0.54	0.66	0.58	
		$\Upsilon(1^3D_1)(10145)$	119	123	0.043	0.05	0.04	
	$\chi_{b1}(2^3P_1)$	$\Upsilon(1^3D_2)$	99	104	1.86	2.31	2.26	
		$\Upsilon(1^3D_1)$	106	110	0.76	0.92	0.84	
	$\chi_{b0}(2^3P_0)$	$\Upsilon(1^3D_1)$	81	87	1.36	1.83	1.85	
	$h_b(2^1P_1)$	$h_{b2}(1^1D_2)(10152)$		104		7.74	7.42	
3P → 2D	$\chi_{b2}(3^3P_2)$	$\Upsilon(2^3D_3)(10442)$	82	97	3.01	5.05	4.69	
		$\Upsilon(2^3D_2)(10438)$	85	101	0.61	1.02	0.89	
		$\Upsilon(2^3D_1)(10432)$	91	107	0.05	0.08	0.07	
	$\chi_{b1}(3^3P_1)$	$\Upsilon(2^3D_2)$	75	86	2.08	3.10	2.98	
		$\Upsilon(2^3D_1)$	81	92	0.86	1.26	1.13	

state	Initial meson	Final meson	E_γ (MeV)		$\Gamma_{\rm thy}$ (keV)			$\Gamma_{\rm expt}$ (keV)
			Ref [30]	$SNR_{0(1)}$	Ref [30]	SNR_0	SNR_1	PDG [18]
3P $\to$ 1D	$\chi_{b2}(3^3\mathrm{P}_2)$	$\Upsilon(1^3\mathrm{D}_3)(10156)$		377	≈ 0	≈ 0	0.05	
		$\Upsilon(1^3\mathrm{D}_2)(10151)$		381	≈ 0	≈ 0	≈ 0	
		$\Upsilon(1^3\mathrm{D}_1)(10145)$		387	≈ 0	≈ 0	≈ 0	
	$\chi_{b1}(3^3\mathrm{P}_1)$	$\Upsilon(1^3\mathrm{D}_2)$		366	≈ 0	≈ 0	0.09	
		$\Upsilon(1^3\mathrm{D}_1)$		372	≈ 0	≈ 0	0.004	
	$\chi_{b0}(3^3\mathrm{P}_0)$	$\Upsilon(1^3\mathrm{D}_1)$		351	≈ 0	≈ 0	0.17	
	$h_b(3^1\mathrm{P}_1)$	$h_{b2}(1^1\mathrm{D}_2)(10152)$		370		≈ 0	0.24	
1D $\to$ 1P	$\Upsilon(1^3\mathrm{D}_3)(10156)$	$\chi_{b2}(1^3\mathrm{P}_2)$	245	240	24.3	26.4	24.5	
	$\Upsilon(1^3\mathrm{D}_2)(10151)$	$\chi_{b2}(1^3\mathrm{P}_2)$	240	236	5.7	6.29	5.87	
		$\chi_{b1}(1^3\mathrm{P}_1)$	261	255	22.0	23.8	19.8	
	$\Upsilon\mathrm{D}_1)(10145)$	$\chi_{b2}(1^3\mathrm{P}_2)$	233	230	0.58	0.65	0.61	
		$\chi_{b1}(1^3\mathrm{P}_1)$	254	249	11.3	12.3	10.3	
		$\chi_{b0}(1^3\mathrm{P}_0)$	285	282	21.4	23.6	16.7	
	$h_{b2}(1^1\mathrm{D}_2)(10152)$	$h_b(1^1\mathrm{P}_1)$		246		42.3	36.5	
2D $\to$ 2P	$\Upsilon(2^3\mathrm{D}_3)(10442)$	$\chi_{b2}(2^3\mathrm{P}_2)$	174	172	16.3	18.0	15.9	
	$\Upsilon(2^3\mathrm{D}_2)(10438)$	$\chi_{b2}(2^3\mathrm{P}_2)$	171	168	3.83	4.17	3.82	
		$\chi_{b1}(2^3\mathrm{P}_1)$	183	181	14.2	15.7	12.1	
	$\Upsilon\mathrm{D}_1)(10432)$	$\chi_{b2}(2^3\mathrm{P}_2)$	165	162	0.38	0.42	0.39	
		$\chi_{b1}(2^3\mathrm{P}_1)$	178	175	7.2	7.87	6.35	
		$\chi_{b0}(2^3\mathrm{P}_0)$	202	198	14.2	15.1	9.49	
	$h_{b2}(2^1\mathrm{D}_2)(10439)$	$h_b(2^1\mathrm{P}_1)$		181		31.3	25.4	
2D $\to$ 1P	$\Upsilon(1^3\mathrm{D}_3)$	$\chi_{b2}(1^3\mathrm{P}_2)$	518	517	3.94	4.01	3.73	
	$\Upsilon(1^3\mathrm{D}_2)$	$\chi_{b2}(1^3\mathrm{P}_2)$	514	513	0.97	0.98	0.68	
		$\chi_{b1}(1^3\mathrm{P}_1)$	534	531	3.25	3.26	4.46	
	$\Upsilon\mathrm{D}_1)$	$\chi_{b2}(1^3\mathrm{P}_2)$	509	507	0.10	0.11	0.05	
		$\chi_{b1}(1^3\mathrm{P}_1)$	529	525	1.75	1.76	1.87	
		$\chi_{b0}(1^3\mathrm{P}_0)$	559	557	2.76	2.79	6.20	
	$h_{b2}(1^1\mathrm{D}_2)$	$h_b(1^1\mathrm{P}_1)$		522		6.19	7.30	

TABLE V: Hyperfine and fine splittings in units of MeV for charmonium and bottomonium in our model. Here σ is $1.362\,GeV$ for charmonium and $3.3\,GeV$ for bottomonium. The experimental values are the mass differences of the corresponding charmonium and bottomonium states taken from PDG [18]. Results of Ref.[36] and Ref.[4] are listed for comparison.

State	Charmonium		Bottomonium			
	Ours	Exp	Ours	Ref.[36]	Ref.[4]	Exp
$1^3S_1 - 1^1S_0$	118	116.6 ± 1.2	71	87	60	$71.4^{+2.3}_{-3.1} \pm 3.7$
$2^3S_1 - 2^1S_0$	50	52 ± 4	29	44	20	
$3^3S_1 - 3^1S_0$	31		21	41	10	
$1^3P_2 - 1^3P_1$	44	45.54 ± 0.11	21	22	20	19.43 ± 0.57
$1^3P_1 - 1^3P_0$	77	95.91 ± 0.32	32	30	30	33.34 ± 0.66
$2^3P_2 - 2^3P_1$	36		18	18	10	13.19 ± 0.77
$2^3P_1 - 2^3P_0$	59		25	25	20	22.96 ± 0.84
$3^3P_2 - 3^3P_1$	30		16			
$3^3P_1 - 3^3P_0$	47		22			